Calculus
Review for Q1 Exam 2

Key

Do Now:

Find the derivative: $g(x) = (\sin x)(4\sqrt{x})$

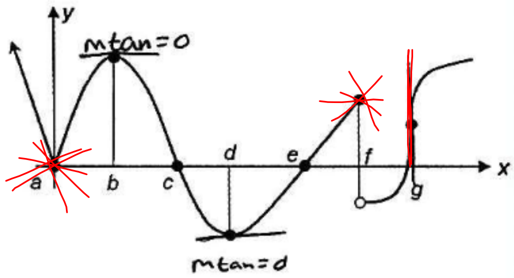
$(\sin x)(4x^{\frac{1}{2}})$

$g'(x) = (\sin x)(2x^{-\frac{1}{2}}) + 4x^{\frac{1}{2}}(\cos x)$

$g'(x) = \frac{2}{\sqrt{x}} \sin x + 4\sqrt{x} \cos x$

$g'(x) = \frac{2}{\sqrt{x}} \sin x + 4\cos x\sqrt{x}$

1. Use the graph below to determine all x-values where the function is **not** differentiable.



a, f, g

Jump, corner, cusp, endpts, vertical line

2. Find the x-coordinates of all points on the graph of $y = x^3 - 5x^2 - 8x + 9$ at which the tangent line is horizontal.

m = 0
slope = 0
derivative = 0

1) Set $y' = 0$
2) Solve for x

$y' = 3x^2 - 10x - 8$

$3x^2 - 10x - 8 = 0$

AC: −24
b: −10
−12 & 2

$3x^2 - 12x + 2x - 8 = 0$

$3x(x-4) + 2(x-4) = 0$

$(3x+2)(x-4) = 0$

$x = -2/3 \quad x = 4$

3. Use the limit definition of a derivative to calculate the derivative of $f(x) = 5 - 2x^2$

$$f'(x) = \lim_{h \to 0} \frac{5 - 2(x+h)^2 - (5 - 2x^2)}{h}$$

$$= \frac{5 - 2(x^2 + 2xh + h^2) - 5 + 2x^2}{h}$$

$$= \frac{\cancel{5} - \cancel{2x^2} - 4xh - 2h^2 \cancel{- 5} + \cancel{2x^2}}{h}$$

$$\frac{-4xh - 2h^2}{h}$$

$$\frac{-2\cancel{h}(2x - h)}{\cancel{h}} = \boxed{-4x}$$

4. Find the derivative slope of the tangent line to the graph of the function $y = 5x^2 - 3x$ when $x = -1$

1) find y'
2) find m tan line
extra not asked for Eq of tan line 3) Plug $x = -1$ into

$y' = 10x - 3$

$y' = 10(-1) - 3$

$y' = -13$

5. Find $h'(x)$ when $h(x) = 4\sqrt{x} + 5\cos x$

$h(x) = 4x^{\frac{1}{2}} + 5\cos x$

$h'(x) = 2x^{-\frac{1}{2}} + 5(-\sin x)$

$h'(x) = \frac{2}{\sqrt{x}} - 5\sin x$

6. Find the h'(x) if h(x) = (cos(x))($3x^3 - x^2 + 10x + 2$).

u v

$$h'(x) = (\cos x)(9x^2 - 2x + 10) + (3x^3 - x^2 + 10x + 2)(-\sin x)$$

$$h'(x) = \cos x(9x^2 - 2x + 10) - \sin x(3x^3 - x^2 + 10x + 2)$$

7. Use the following table to find y ' $\boxed{\text{at } x = 1}$ if:

f(1)	f ' (1)	g(1)	g ' (1)
3	4	1	-2

y = f(x)g(x)

$$y' = f(x)g'(x) + g(x)f'(x)$$

$$y' = (3)(-2) + (1)(4)$$

$$y' = -6 + 4$$

$$y' = -2$$

8. Find the coordinates of the point(s) where $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$ has horizontal tangents.

$m_{tan} = 0$

Set $f'(x) = 0$

$$f'(x) = x^3 - x^2 - 2x$$

$$x^3 - x^2 - 2x = 0$$

$$x(x^2 - x - 2) = 0$$

$$x(x-2)(x+1) = 0$$

$$x = 0, 2, -1$$

Plug x = 0, 2, -1 into f(x)

(0, 0) (2, -8/3) (-1, 5/12)

$f'(x) = u \cdot v' + v \cdot u'$

9. Find $f'(x)$ if $f(x) = \overset{u}{3x^2}\,\overset{v}{\sin x}$

$f'(x) = 3x^2 \cos x + \sin x\,(6x)$

$f'(x) = 3x^2 \cos x + 6x \sin x$

Find the slope of the tangent line to the function at the given x-value for each of the following:

10) $f(x) = x^2 + 8x + 16 \quad when\ x = -2$

$f'(x) = 2x + 8$

$f'(-2) = 2(-2) + 8$
$= 4$

11) $f(x) = 3x^2 - 4x + 2 \quad when\ x = 2$

$f'(x) = 6x - 4$

$f'(2) = 6(2) - 4$
$= 8$

12) $f(x) = \overset{u}{(3x-5)}\overset{v}{(x^2+9x)} \quad when\ x = 1$

$(3x-5)(2x+9) + (x^2+9x)(3)$

$(3(1)-5)(2(1)+9) + (1^2+9)(3)$

$(-2)(11) + 30)$

$-22 + 30 \rightarrow 8$

13) $f(x)=\sqrt{3x-3}$ *when* $x=4$ (Use the limit definition)

$$f'(x)=\lim_{h\to 0}\frac{\sqrt{3(x+h)-3}-\sqrt{3x-3}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{3(x+h)-3}-\sqrt{3x-3}}{h}\cdot\frac{\sqrt{3(x+h)-3}+\sqrt{3x-3}}{\sqrt{3(x+h)-3}+\sqrt{3x-3}}$$

$$=\lim_{h\to 0}\frac{3(x+h)-3-(3x-3)}{h\sqrt{(3(x+h)-3}+\sqrt{3x-3}}$$

$$=\lim_{h\to 0}\frac{\cancel{3x}+3\cancel{h}\cancel{-3}\cancel{-3x}\cancel{+3}}{\cancel{h}\sqrt{3(x+h)-3}+\sqrt{3x-3}}$$

$$=\lim_{h\to 0}\frac{3}{2\sqrt{3x-3}}=\frac{3}{2\sqrt{3(4)-3}}=\frac{3}{2\sqrt{9}}\to\frac{3}{6}\to\frac{1}{2}$$